Modeling fatigue crack growth behavior in rolled AZ31 magnesium alloy using CTOD based strip yield modeling

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Abstract
Fatigue crack growth behavior of a rolled AZ31 magnesium plate is investigated and modelled in this study. Fatigue crack growth tests were performed on compact tension specimens at load ratios of \( R = 0.1 \) and \( R = 0.7 \) to provide data for a strip-yield based fatigue crack growth model. Minimal differences in crack closure were observed between the two load ratios. Threshold values for the stress intensity factor range were found to be often less than those reported in previous literature. The reason for lower threshold values could be related to the nontraditional compression pre-cracking method used in this study, which was employed in an attempt to produce a more accurate measurement of the fatigue crack threshold. In addition to the long crack fatigue crack growth testing using compact tension specimens, load controlled fatigue tests were conducted on flat, reduced gage specimens at load ratios of \( R = 0.1 \) and \( R = -1.0 \) to study the microstructurally small crack growth behavior and to extend the model capability to predict the microstructurally small crack growth behavior. The reduced gage specimens were found to spend the majority of their life in the crack growth stage. Cracks primarily grew in a planar fashion other than where multiple cracks coalesced. The microstructurally small crack growth data from these experiments were also compared with predictions from the fatigue crack growth model. Crack growth modeling revealed that traditional calculations using plasticity-induced crack closure concepts and an effective stress intensity factor range were not able to predict the microstructurally small crack growth behavior of these specimens. In contrast, computing crack growth rate from crack tip opening displacement was shown to give satisfactory results. Crack opening stresses for the fully reversed tests revealed that the compressive loading largely nullified the effect of the plastic wake on fatigue crack growth.

1. Introduction

Wrought magnesium alloys have been increasingly used in a wide variety of structural components due to low density and relatively high strength. Magnesium alloys are of interest in automotive applications to replace various structural parts made of steel and aluminum alloys to increase fuel efficiency in vehicles. Examples of applications in the automotive vehicle include engine blocks, transfer cases, and gear boxes [1]. Most of these components and structures are under an extended period of repeated loading. Thus, the fatigue behavior of wrought magnesium alloys must be well understood and characterized to avoid structural failure due to cyclic loading.

Wrought magnesium is preferred over cast magnesium because cast magnesium is typically associated with large inclusions [2]. Wrought magnesium is known to be a highly anisotropic material [3]. This anisotropy is due to the strong texture that is formed during the rolling or extrusion process. More specifically, the rolling process ideally aligns the hexagonal closed packed (HCP) grains of the magnesium such that the c-axis of the grains is parallel with the normal direction of the plate [4]. Anisotropy refers to the different plastic deformation mechanisms that can occur depending on the direction of loading on the wrought magnesium with respect to how the grains are oriented. When the c-axis of the HCP grain is loaded in tension, plastic deformation occurs primarily as twinning due to a shear stress caused by the applied stress state in combination with the low critical resolved shear stress needed to induce twinning [3,5]. Twinning also allows for further straining along the c-axis direction. Conversely, compressive loading parallel to the c-axis will force the HCP crystal to be...
primarily under the influence of slip systems [3]. For an ideal rolled magnesium plate, compression in the rolling or transverse direction and tension in the normal direction will primarily cause twinning. Otherwise, tension in the rolling or transverse direction and compression in the normal direction will primarily cause slip. Twinning initiates at a significantly lower stress compared to slip in magnesium alloys [6]. Furthermore, when magnesium is monotonically loaded so that plastic deformation begins as twinning, the material experiences increased hardening until the material is close to fracture. When plastic deformation begins as slip under monotonic loading, various degrees of hardening can initially be observed, but the softening rates tend to level out as the material is strained further [6].

Under fatigue conditions, deformation twinning can be distinguished on a stress-strain hysteresis loop by a characteristic plateau, provided the stress and strain are high enough during the specific reversal [3]. Whether the tensile or compressive reversal activates twinning deformation is dependent on the texture of the material in combination with the direction of loading, similar to the monotonic conditions. The fatigue life of metallic materials consists of crack incubation, small crack growth, and long crack growth. The microstructurally small crack growth and long crack growth regimes were previously shown [7] to constitute the majority of the fatigue lifetime in wrought AZ31 magnesium. Cracks that are still under the influence of microstructure will be referred to interchangeably in this study as microstructurally small cracks and small cracks. Because crack growth comprises the majority of the fatigue life for this alloy, modeling microstructurally small crack growth behavior is critical for accurately predicting the fatigue life of a component.

However, microstructurally small crack growth behavior is particularly difficult to predict due to several factors. First, small cracks typically possess higher crack growth rates than would be predicted by linear elastic fracture mechanics [8]. Secondly, because small cracks lack a large amount of plastic wake behind the crack tip, they typically stay open longer and grow faster. Thirdly, small cracks are also heavily influenced by the local microstructure ahead of the crack tip [9,10]. They are under the influence of microstructure until the crack has developed enough plasticity such that the crack tip plastic zone is comprised of several grains, which typically happens when the crack is several times the size of the surrounding grains [11]. Grain boundaries can hinder or completely arrest the advancement of microstructurally small cracks as the cracks grow into the boundaries [12]. The deflection or deceleration of small cracks due to grain boundaries is a major factor with regard to the amount of crack closure that a small crack encounters. Furthermore, microstructurally small crack growth rates exhibit little to no deceleration if the grain into which the microstructurally small crack is entering has a similar orientation to the grain it currently occupies [9].

For rolled AZ31, crack propagation rates for cracks growing orthogonal to [1010] planes are significantly faster than the crack propagation rates for cracks growing orthogonal to the (0001) plane [13]. Yin et al. [14] observed that twinning around the crack tip during fatigue loading can affect the roughness of the crack path in an AZ31 magnesium alloy. When the crack is small and lacks a large amount of plasticity around the crack tip, twins formed at the crack tip do not typically detwin and the crack remains in compressive contact for a longer period of time. This was also considered to leave residual twins in the plastic wake as the crack progressed. Wu et al. [15] showed that texture far away from a long fatigue crack that has developed a sizable plastic zone was similar to texture in the wake of a fatigue crack in terms of twinning in a rolled AZ31 plate. This could be because of the large plastic zone that was developed at the crack tip allowing for complete to almost complete detwinning as the crack grew through the material. Rolled AZ31 magnesium has been observed to exhibit short and long crack growth rates which increase somewhat linearly with increased crack length under uniaxial fatigue loading [7]. The microstructurally small crack growth rate may have a linear relationship with the crack tip opening displacement [11,16].

McDowell et al. [11] developed a fatigue model for cast A356-T6 alloy that partitions the fatigue life into distinct stages of crack incubation, microstructurally small crack growth, physically small crack growth and long crack growth. This model is also able to incorporate microstructural features for more accurate crack growth predictions. The McDowell et al. approach has been utilized to model the fatigue behavior of various types of magnesium alloys [17–21]. While this model has been shown to predict fatigue life and crack growth quite well, the model is laborious to implement, and much material characterization must take place before the model can be used. This model calculates microstructurally small crack growth as a function of crack tip opening displacement range (ACTOD). To predict crack growth beyond the small crack range, an effective stress intensity factor ($K_{eff}$) is used in their approach and the effects of plasticity-induced fatigue crack closure are considered using FASTRAN, a modified strip-yield model developed by Newman [22].

The purpose of this research is to investigate how well microstructurally small crack growth in a rolled AZ31 magnesium

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**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$a$</td>
<td>crack depth</td>
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<tr>
<td>$A$</td>
<td>deepest point of crack penetration for crack growth modeling</td>
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<tr>
<td>$B$</td>
<td>free surface location for crack growth modeling</td>
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<td>$c$</td>
<td>crack length</td>
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<td>Paris law constant</td>
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plate can be predicted using a strip-yield based fatigue crack growth model developed previously for semi-elliptical surface cracks [23]. The model is an extension of the modified strip-yield model originally developed by Newman, that has been able to successfully predict crack growth behavior for aluminum, titanium, and steel alloys [22–27]. For model input, stress intensity factor range versus crack growth rate data gathered from compact tension (CT) specimens is used. The model uses strip-yield based computations to obtain values for crack front plastic zone size, crack tip opening displacements along the crack front, and crack opening stresses associated with the plastic wake behind the crack front under cyclic loading. To quantify crack growth, specifically microstructurally small crack growth, for this material, a \( D_{\text{CTOD}} \) criterion is implemented and compared with a more traditional, effective stress intensity factor approach using the crack opening stresses. As microstructurally small cracks exhibit a lack of crack closure and are open more often [28], the \( D_{\text{CTOD}} \) criterion may reveal itself to be a more suitable parameter for quantifying microstructurally small crack growth behavior. In this investigation, different test configurations are employed to capture microstructurally small crack growth and long crack growth behaviors of a rolled AZ31 magnesium plate.

2. Material and experimental procedures

2.1. Material and specimens

The material used in this investigation was a rolled AZ31 magnesium tooling plate with a nominal thickness of 3/4” (19.05 mm). The plate was thermally rolled with no post process heat treatment. Table 1 lists the element composition in atomic weight of the as-received plate. Two specimen configurations were cut from the plate for fatigue testing such that loading took place in the rolling direction of the plate, and crack growth took place in the transverse direction. Fig. 1 shows the specimen drawings for the microstructurally small crack growth and compact tension specimens. The microstructurally small crack growth specimen used was a flat reduced gage section design with a very slight notch machined into the center of the gage section on one side as designed in a previous study [21,29]. The slight notch in the gage section was machined to ensure that crack nucleation and growth would occur in that particular area during testing without exceeding a stress concentration of 1.07. The slight notch was also useful for reducing the amount of surface replication material and microscopy observation time needed.

Table 1

<table>
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<tr>
<th>Element</th>
<th>Al</th>
<th>Zn</th>
<th>Mn</th>
<th>Ni</th>
<th>Cu</th>
<th>Fe</th>
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<td>0.99</td>
<td>0.262</td>
<td>0.05</td>
<td>&gt;0.048</td>
<td>0.04</td>
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Fig. 1. Specimen configuration for (a) microstructurally small crack growth and (b) compact tension specimens (all dimensions in millimeters). Pictures to the right depict how the specimens were cut out of the rolled plate with respect to orientation.
Compact tension specimens were used to capture the long crack growth behavior. The specimen was designed according to ASTM standard E647 [30]. After machining, all microstructurally small crack specimens were polished along the loading direction and all CT specimens were polished in the crack growth direction to ensure all machining marks were removed. Several monotonic specimens were also machined so that monotonic properties could be found. Cylindrical tension specimens with uniform gage section with a gage diameter of 6.35 mm and a gage length of 18 mm were used to conduct tension tests. The compression sample, designed according to ASTM standard E909 [31], possessed a diameter of 13 mm and a length of 25 mm.

2.2. Crack growth test procedures

Microstructurally small crack growth specimens were tested using a 25 kN uniaxial servo hydraulic load frame at a maximum stress of 110 and 115 MPa and load ratios of R = 0.1 and R = −1.0. Preliminary fatigue tests revealed that these two stress levels possessed small amounts of scatter concerning life to failure. Since the life to failure was fairly predictable, a suitable surface replication schedule could be implemented. Crack growth measurements were performed using Repliset® which has been shown to capture cracks as small as 10 μm [32]. Repliset® is a two part silicon-epoxy compound that is used to create a negative replication of the surface that the compound is applied to. Although between 12 and 36 replications were taken for each test specimen, not all data points were used for microstructurally small crack growth modeling due to the short crack growth regime being defined in this study as 300 μm and under. The specimens were held at a small tensile load well inside the elastic stress region of the material while the Repliset® was applied to the specimens for the purpose of ensuring the Repliset® better penetrated the crack depth. Imaging was performed on the replications using a scanning electron microscope (SEM). Under the SEM for each specimen, a dominant crack was located and measured on the last replication. For the same specimen, the dominant crack was located on each previous replication and measured until the dominant crack was no longer visible. To identify and keep track of the dominant crack, different methods had to be used to account for crack interaction. For some specimens, a single, large, isolated crack was located on the last replication and deemed to be the dominant crack. Other specimens still had a large, single crack, but the crack was surrounded by other smaller cracks, and therefore, it was most likely under the influence of those smaller cracks. For the former situation, the crack was easily measured on each replication. For the latter situation, a judgment had to be made on how to handle crack coalescence. The effect of crack coalescence on crack growth rates was simply handled by allowing the micro-sized cracks that came in contact with the dominant crack to be added in with the dominant crack length. Life to crack incubation for each specimen was defined to be when the dominant crack could no longer be found.

Long crack tests were also conducted on CT specimens to obtain crack growth rate versus stress intensity factor range data. These tests involved pre-cracking specimens using compression fatigue loading [33]. Following compressive pre-cracking, the specimens were either run at constant amplitude (CPCA) loading or the fixed load ratio load reduction (CPLR) method according to ASTM E647 [30] to generate the needed ΔK versus crack growth rate curves. The load reduction method involves reducing the load until the crack growth rate decreases to a value of 10−10 m/cycle and then defining the ΔK value at which that particular rate occurs as the fatigue crack growth rate threshold. Pre-cracking using compressive fatigue loading served the purpose of ensuring no residual stress near the crack tip exists when the load reduction begins. Compact tension tests were conducted at 18 Hz using load ratios of R = 0.1 and R = 0.7. The crack growth data is required for crack growth predictions using the strip-yield model. For the sake of comparison, traditional tensile pre-cracking followed by constant amplitude (TPCA) loading on CT specimens was also conducted.

3. Experimental results and discussion

3.1. Microstructural characterization

The microstructure was studied in the hopes of finding a relationship between the crack growth behavior and microstructural characteristics. Two samples were removed from the unstressed region of each CT specimen and cold mounted with the rolling direction of the material exposed on one sample and the transverse direction of the specimen exposed on the other. The samples were first ground using progressively finer grits, ending with a 4000 μm grit, and then electropolished to obtain a near mirror finish. Electron backscatter diffraction (EBSD) was performed on each sample to determine how the basal (0001) and prismatic {10 10} planes were oriented in the specimen with respect to the rolling and transverse directions.

Fig. 2 displays the pole figures from one CT specimen with respect to the rolling direction. Pole figures display a two dimensional representation of the density of a specific plane from a specific direction. As the color on the circle goes from blue to red, the density of that specific plane increases.1 If viewed down the rolling direction, texture of a rolled AZ31 plate is ideally composed of a strong intensity of the (0001) plane perpendicular to the normal direction of the plate with the {10 10} planes parallel with the normal direction of the plate. Similar results were obtained for all of the samples analyzed with the EBSD scan. The implications of this texture were briefly discussed in the introduction and stated to have negative effects on the crack growth behavior if crack growth took place in the transverse or rolling direction of the rolled plate [13]. It has been empirically found that cracks grow much faster normal to the prismatic plane of the HCP grain (rolling and transverse directions in this plate) as opposed to parallel to the prismatic plane of the HCP grain (normal direction in this plate) [13]. Both the CT and reduced gage specimens were machined such that crack growth took place in the transverse direction of the plate.

Initial imaging on the optical microscope revealed the abundant presence of intermetallic particles. Investigation of the intermetallic particles using energy dispersive spectroscopy revealed that the particles had an aluminum-manganese composition, as also reported in a previous study considering magnesium alloys [7]. The appearance of the particles typically took on very distinct shapes, as shown in Fig. 3. The particles, in some cases, served as locations of crack incubation by causing stress concentrations. However, a correlation between small crack growth specimens with particles at the crack incubation site and fatigue life could not be obtained. As-received samples were taken from the rolling and transverse directions of the plate to determine grain size. Samples were polished to a near mirror finish and then etched with an acetic-picral solution to reveal grain boundaries. Fig. 4 shows the results of the etched material. A fairly equiaxed grain structure was revealed on the surface of the material for both directions. The average grain size of the rolled plate was 9–10 μm, calculated according to the linear intercept method in ASTM standard E-112 [34]. Some grains were calculated to have an equivalent diameter as large as 29 μm and as small as 3.1 μm, and the standard deviation was calculated to be 4.52 μm.

1 For interpretation of color in Fig. 2, the reader is referred to the web version of this article.
3.2. Monotonic and fatigue results

3.2.1. Monotonic behavior

The results of the monotonic tests on the tension and compression specimens are discussed in this section. Fig. 5 displays the results from the tension and compression monotonic tests, and several key values from this figure are listed in Table 2. After yielding, the compression curve displays a large amount of hardening compared to the tensile curve. Because twin systems are so easily activated under compressive loading for this texture, the amount of twinned grains quickly increased in the material following yielding. The increased amount of twinned grains allows for more slip activation [35,36]. The hardening that occurs shortly after compressive yielding is considered to be a result of these dislocations piling up. While the compression and tension curves have the same initial modulus of elasticity, yielding occurs at a lower stress in compression due to the lower critical resolved shear stress associated with twinning [37]. The yielding stress in tension, 159 MPa, is almost twice the amount of compressive stress at yielding, 76 MPa. Twin activation is related to grain size in a material, where larger grains allow for more twinning. With the relatively small average grain size seen in this effort, more hardening will possibly occur with larger grain sizes. Although the yield stress is lower in compression, the compressive ultimate stress is much higher than tension due to the twinning phenomenon causing hardening under compressive loading. The ultimate compressive stress averaged to be approximately 367 MPa, while the average ultimate tensile stress was approximately 263 MPa.

3.2.2. Fatigue crack growth behavior

Fig. 6 shows the fatigue crack growth results for the CT specimens. Values for $\Delta K_text{th}$ under $R = 0.1$ loading in this investigation averaged approximately 1.37 MPa$\sqrt{m}$, and previously published values were found to be approximately 1.90 MPa$\sqrt{m}$ [29]. The threshold values reported in [29] were for $R = 0.05$ loading, and the values recorded from this study were for $R = 0.1$ loading. For $R = 0.7$ loading, the $\Delta K_text{th}$ value in this study was found to be approximately 0.69 MPa$\sqrt{m}$. Zheng et al. [38] conducted tests on CT specimens from extruded AZ31 with a similar orientation and
observed a change in $D$ that little crack closure exists when comparing $R = 0.1$ loading to the load ratio from 0.1 to 0.75. In this study, the difference in the scatter is still relatively small.

The amount of crack incubation for microstructurally small cracks in the fully rolled. The compression pre-cracking load reduction method used in this investigation seemingly revealed lower $AK_{th}$ values than seen in most literature. The ultimate goal of gathering long crack data using CT specimens was to collect needed input for the strip-yield model presented later. The strip-yield based fatigue crack growth model used here requires fatigue crack growth rate as a function of effective stress intensity factor range. This was assumed equivalent to collecting data under $R = 0.7$ where little or no crack closure exists. The lower end of the $R = 0.7$ curve data was taken and approximated using six separate power law crack growth equations that were functions of $AK_{th}$. Using these six equations, the first step of implementing the strip-yield model for this material was completed. These equations are represented as a dashed line in Fig. 6.

To compare experimental microstructurally small crack growth data with the model predictions, load control tests were conducted on the reduced gage section specimen with a very slight notch in the center. After the surface replications of the reduced gage specimens were analyzed via SEM, the incubation and crack growth behavior of those tests were assessed from the replication analysis. As previously mentioned, between 12 and 36 replications were obtained for total crack growth of a given specimen. Only a fraction of these were considered for microstructurally small crack data in most cases due to the microstructurally small crack growth regime being limited to 300 $\mu$m in this study. Small cracks in the plate were most likely beyond the effect of microstructural influence at this crack length limit considering the average grain size was experimentally found to be a little less than 10 $\mu$m. Obtaining data for crack lengths beyond 1 mm for the load control specimens was typically not possible due to imminent fracture occurring after cyclic loading beyond this crack length. To study the long crack growth using the same replication method, a different fatigue specimen is needed. In addition, the experimentalist must run the desired number of cycles, followed by 10–20 min for the replication to dry in order to capture a single data point using replications. Accordingly, a large time investment is needed if a large number of replications is taken. Although not investigated, the authors agree with Jordon et al.'s [32] conclusion that the presence of the utilized replication material did not affect the overall crack growth rate of the material.

As seen from the replications shown in Fig. 7, minimal crack deflection took place for both loading ratios. Because crack growth for this material almost always took place in the transverse direction with loading in the rolling direction of the plate, minimal to no crack deflection took place. Crack growth through this grain orientation has been shown only to be subject to significant deflection when the crack encounters a large cluster of smaller grains [38]. Besides creating a slight stress concentration in the presence of cracks, the machined notch was deemed not to influence the behavior of crack growth in any significant manner. The minimal amount of non-planar crack growth that was observed via replications occurred at the location where two cracks were coalescing. A good example of this non-planar crack growth can be seen in Fig. 7 (a) at the edges of the individual cracks. The apparent voids seen in Fig. 7(b) are thought to be a result of the imperfect bonding of the replication to the surface and not a direct replication of the surface. In addition, intermetallic particles often served as locations of crack incubation in the material.

The amount of crack incubation for microstructurally small cracks also differed between load ratios. Minimal crack incubation was observed for the microstructurally small cracks in the fully reversed specimens, while more crack incubation was seen under $R = 0.1$ loading, as shown in Fig. 7. It is possible that higher $R$ values lead to higher levels of crack incubation and coalescence in this material, but that is beyond the scope of this study. Table 3 dis-

<table>
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<th>Tensile and compressive monotonic properties of rolled AZ31 magnesium.</th>
<th>Tensile</th>
<th>Compressive</th>
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<tr>
<td>Yield strength (MPa)</td>
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<td>Ultimate strength (MPa)</td>
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<td>Modulus of elasticity (GPa)</td>
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Fig. 6. Crack growth results from CT specimens. Both compression pre-cracking load reduction (CPLR) and compression pre-cracking constant amplitude (CPCA) results are displayed in conjunction with tensile pre-cracking constant amplitude (TCPA) test results. The dashed lines represent the Paris law equations used to calculate $dA/dN$ in the strip-yield model.

texture to the specimens tested in this study, and the threshold for tests run under $R = 0.75$ loading for the extruded material was reported to be 1.30 MPa/$\sqrt{m}$. Zheng et al. also found that $AK_{th}$ values had extremely small amounts of scatter when the load ratio was changed from 0.1 to 0.75. It is important to note that this amount of scatter was only for the specimens tested such that crack growth took place in the extrusion direction, which is perpendicular to the prismatic planes of the grains within the specimen. Threshold results reported by Zheng at $R = 0.1$ compared well with data presented here. This leads to the consideration that the compression pre-cracking load reduction method does not always produce lower thresholds during testing.

From Fig. 6, an increased $R$ value produced a lower threshold and a lower $AK$ value when final fracture was observed. The crack growth rate values for the two load ratios began to merge as the stress intensity factor approaches 2.20 MPa/$\sqrt{m}$, and the two curves begin to merge fully for $AK = 4.50$ MPa/$\sqrt{m}$. This suggests that little crack closure exists when comparing $R = 0.1$ loading to $R = 0.7$ loading for $AK > 4.50$ MPa/$\sqrt{m}$. Zheng et al. [38] only observed a change in $AK_{th}$ value of 0.04 MPa/$\sqrt{m}$ when changing the load ratio from 0.1 to 0.75. In this study, the difference in $AK_{th}$ value when changing from a load ratio of 0.1 to 0.7 was 0.68 MPa/$\sqrt{m}$. While more $AK_{th}$ scatter was seen in this study, the scatter is still relatively small.

The differences between $AK_{th}$ values observed here and those published elsewhere could be because of the differences in how $AK_{th}$ was obtained. The data referenced for the $R = 0.1$ loading [29] did not use compression pre-cracking methods, but rather conventional tensile pre-cracking. The reference for $R = 0.75$ loading [38] also did not use the compression pre-cracking method when capturing stress intensity factor range versus crack growth rate, and the material in that study was extruded rather than rolled. The compression pre-cracking load reduction method used in this investigation seemingly revealed lower $AK_{th}$ values than seen in most literature. The ultimate goal of gathering long crack data using CT specimens was to collect needed input for the strip-yield model presented later. The strip-yield based fatigue crack growth model used here requires fatigue crack growth rate as a function of effective stress intensity factor range. This was assumed equivalent to collecting data under $R = 0.7$ where little or no crack closure exists. The lower end of the $R = 0.7$ curve data was taken and approximated using six separate power law crack growth equations that were functions of $AK_{th}$. Using these six equations, the first step of implementing the strip-yield model for this material was completed. These equations are represented as a dashed line in Fig. 6.

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</tr>
<tr>
<td>Modulus of elasticity (GPa)</td>
<td>44.1</td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>76.0</td>
</tr>
<tr>
<td>Ultimate strength (MPa)</td>
<td>366.8</td>
</tr>
<tr>
<td>Elongation to failure (%)</td>
<td>NA</td>
</tr>
<tr>
<td>Modulus of elasticity (GPa)</td>
<td>44.1</td>
</tr>
</tbody>
</table>
plays the cycles to failure and total life spent in crack growth for each small crack growth specimen tested. It is clearly seen that the majority of specimens spent most of the fatigue life in the crack growth regime. This is consistent with a previous study conducted by Ishihara et al. [7]. It is important to note that all specimens had a very short fatigue life (less than 1.3 × 10^4 cycles) with very rapid fatigue crack growth.

Under R = 1.0 loading, crack growth calculations were not thought to be under the effects of crack interactions. However, the dominant cracks of specimens run under R = 0.1 loading were almost constantly under the effect of crack interactions due to the high amount of crack incubation occurring on the surface of those specimens. Concerning general crack growth, cracks growing coplanar or close to coplanar will have an effect on one another. As two coplanar cracks approach one another, the crack growth rate increases for both cracks [39]. At some point, the two coplanar cracks will grow close enough such that the stress intensity factor will increase for both cracks, which further increases the crack growth rate of each crack. There is also the issue of cracks tending to “curve” toward the interacting cracks when the interacting cracks aren’t coplanar. The observation that cracks curve as such indicate that not only will a mode I stress intensity factor be present, but also a mode II stress intensity factor is affecting the crack [39,40]. This is expected to increase crack growth rates further. Based on this discussion, crack growth rates of the R = 0.1 data will be accelerated to some degree because of the crack interactions. However, altering the utilized strip-yield model to incorporate crack interaction effects is beyond the scope of this study.

4. Crack growth modeling and results

A previously developed strip-yield based fatigue crack growth model was used in this study to predict the growth of microstructurally small semi-elliptical fatigue cracks [23,24,27]. This particular model has not been used previously to predict small crack growth as will be attempted in this paper. As illustrated in Fig. 8, the surface flaw under a remote traction, \( \sigma(x,y) \), is synthesized using two groups of through-crack strip-yield models and an unknown shear traction, \( P(x,y) \). The slice synthesis methodology allows for computation of surface flaw stress intensity factors, crack surface displacements, and crack front plastic deformations after the shear traction has been determined using displacement compatibility [23,27]. The slice synthesis methodology was first introduced by Fujimoto [41] and Saff et al. [42] for stress intensity factor computation. More recently, it was used by Zhao et al. [43,44] to compute surface crack and corner crack stress intensity factors and crack opening displacements. Fatigue crack growth was simulated by extending the crack front incrementally and computing the corresponding plastic wake behind the growing crack front as originally proposed by Newman in his two dimensional modi-

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Table 3
Stress life fatigue results obtained from the small crack growth specimens.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>R</th>
<th>( \sigma_a ) (MPa)</th>
<th>( N_{inc}/N_f )</th>
<th>( N_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>–1.0</td>
<td>115</td>
<td>0.54</td>
<td>6500</td>
</tr>
<tr>
<td>18</td>
<td>–1.0</td>
<td>115</td>
<td>0.19</td>
<td>5384</td>
</tr>
<tr>
<td>19</td>
<td>–1.0</td>
<td>110</td>
<td>0.54</td>
<td>10,158</td>
</tr>
<tr>
<td>20</td>
<td>–1.0</td>
<td>110</td>
<td>0.44</td>
<td>12,563</td>
</tr>
<tr>
<td>21</td>
<td>0.1</td>
<td>115</td>
<td>0.37</td>
<td>6030</td>
</tr>
<tr>
<td>22</td>
<td>0.1</td>
<td>115</td>
<td>0.32</td>
<td>6283</td>
</tr>
<tr>
<td>23</td>
<td>0.1</td>
<td>115</td>
<td>0.30</td>
<td>5271</td>
</tr>
<tr>
<td>24</td>
<td>0.1</td>
<td>110</td>
<td>0.21</td>
<td>6411</td>
</tr>
<tr>
<td>25</td>
<td>0.1</td>
<td>110</td>
<td>0.33</td>
<td>7550</td>
</tr>
<tr>
<td>26</td>
<td>0.1</td>
<td>110</td>
<td>0.09</td>
<td>5625</td>
</tr>
</tbody>
</table>

---

Fig. 7. Replications of specimens under (a) R = –1.0 loading and (b) R = 0.1 loading. Specimens tested with an R value of 0.1 consistently showed an increased amount of crack incubation relative to specimens tested with an R value of –1.0.
fied strip-yield model [22]. Fatigue crack growth is predicted for the deepest point of penetration (A) and the free surface (B). Plasticity induced crack closure is considered, and the crack tip opening displacement and the crack opening stress at points A and B are computed. Further details regarding the strip-yield based slice synthesis model for calculation of the plastic wake, shear traction, and cyclic crack tip opening displacement can be found in Ref. [23,24,27].

The model requires specification of an initial flaw size a and 2c, a modulus of elasticity E, and tensile yield stress, σy. An initial circular flaw with a 25 μm radius was specified in the model to reflect one of the smallest crack lengths found experimentally. The model also requires tensile constraint factors, k, such that, under maximum loading, the stress in the plastic zone is aσy. A distinct value is required for points A and B, where an a value of 3 represents plane strain and a value of 1 represents plane stress. To approximate the significantly lower compressive yield stress exhibited by the AZ31 magnesium rolled plate, a compressive constraint factor, β, was added as a new model input such that under the minimum loading, compressive yielding along the crack wake is achieved at a stress level βσy, where \( β < 1 \).

Spring (c) and primary (a) slice tension constraint factors were assigned a value of 2.5 for this study as a small flaw is predominantly under plane strain conditions. Both the primary and spring slice compression constraint factors were assigned a value of 0.5. This particular value for the compressive constraint factor was chosen to represent that the compressive yield strength of the material is approximately half of the tensile yield strength. The purpose of these additional compressive constraint factors is to alter the plastic wake left by compressive loading. As compression loading is applied, the plastic wake experiences a reduction in size. This decrease in plastic wake size reduces the amount of crack closure that occurs and the subsequent crack opening load. Because this particular magnesium alloy and texture yields so easily in compression, extensive crack wake deformation is expected under the minimum loading in the fatigue cycle.

To calculate the crack opening stress normalized by maximum stress, \( \sigma_o/\sigma_{\text{max}} \) the model takes into account the current cyclic stress applied and the plastic wake left behind the crack tip by the previous crack growth. These opening stresses are the important foundation for the prediction of crack growth. Additional details regarding the strip-yield model can be found in [23,24,27]. As expected, the predicted crack opening stresses differed drastically between the tests run at \( R = 0.1 \) loading and \( R = -1.0 \) loading. Fig. 9 displays \( \sigma_o/\sigma_{\text{max}} \) at point B of the semi-elliptical flaw for all stress levels and loading ratios. Crack opening stresses for \( R = -1.0 \) loading were calculated to be nearly zero. This implies that the crack was completely open during the tensile portion of the cyclic loading with the crack seemingly following linear elastic fracture mechanics principles. The non-existent crack opening stress is due to the combination of the low compressive yield strength and repeated compressive loading. Because the crack is open for a larger portion of the tensile cycle, specimens run under \( R = -1.0 \) loading will experience faster crack growth. Normalized crack opening stresses for \( R = 0.1 \) loading are also displayed in Fig. 9. At a crack length of 25 μm, \( \sigma_o/\sigma_{\text{max}} \) is around 0.135 and rising for both stress levels. The stress begins to level out as the crack reaches approximately 40 μm. Although the crack opening stresses fluctuate at this point, they do not go higher than 0.175 or lower than 0.15 for the remainder of the microstructurally small crack regime.

The model, as originally developed, assumed crack growth rate was governed by the effective stress intensity factors computed at points A and B from the computed crack opening stresses. A required model input was a crack growth rate versus range \( \Delta K \) curve at large R values such that the data was closure free. Initial calculations were performed to predict crack growth and were deemed unsatisfactory. Fig. 10 displays the crack growth rate prediction plotted against experimental data using the effective stress intensity factor range. Using the Paris equation with an effective stress intensity factor range for microstructurally small crack growth, the predicted crack growth rates were at least an order of magnitude slower than the experimental data. It is important to note that the 1.07 stress concentration value was not taken into account at any point in the crack growth rate modeling.

It was obvious at this point that another method of calculating crack growth rate must be utilized to better match the measured crack growth rates. According to a classic publication by McClintock [45], extracting crack growth rate from \( \Delta C T O D \) was a viable method for predicting crack growth behavior. He showed that crack growth calculated from \( \Delta C T O D \) predicted faster crack growth than that from the Paris equation. Models using \( \Delta C T O D \) to predict crack growth behavior are known as geometrical models [16]. The reasoning for this naming stems from the geometrical relationship that exists between striation spacing, crack tip blunting, and crack growth rates, which constitutes \( \Delta C T O D \) modeling [16].\( \Delta C T O D \) based models have also been successfully used by Lankford [8], Chan et al. [46], and Chan [47], McDowell et al. [11] utilized this concept and derived a simple equation to relate \( \Delta C T O D \) and crack growth rate for microstructurally small cracks as shown in Eq. (1).

\[
\frac{dc}{dN} = G(\Delta C T O D - \Delta C T O D_{th})
\]  

(1)

The material constant G scales down the output from \( \Delta C T O D \). The strip-yield model readily computes crack front displacement as a part of the process for computing crack opening stress, so extracting \( \Delta C T O D \) for crack growth prediction was a fairly trivial task. The term \( \Delta C T O D_{th} \) was taken to be the Burgers vector for the metal being considered [11]. The model extends the crack once the \( \Delta C T O D \) grows beyond the \( \Delta C T O D_{th} \). For magnesium, this threshold value is 3.2 \times 10^{-4} \text{ μm} [48]. G was assigned a value of 0.32 [21]. Fig. 11 shows that crack growth rates calculated using Eq. (1) correlates reasonably well with the experimental data with the given parameters. The figure gives crack growth rate as a function of crack length for all tests while the crack length was less than 300 μm. While the experimental \( R = -1.0 \) and \( R = 0.1 \) data possessed similar crack growth rates at the same crack length, the model predicted faster crack growth for the \( R = -1.0 \) loading. This higher predicted crack growth rate for \( R = -1.0 \) loading is a direct result of the plastic wake reduction previously discussed. Sensitivity of the microstructurally small crack growth behavior model to compressive constraint factors was evaluated by altering the values of compressive constraint factors between 0.45 and
0.55. Changing these constraint factors only slightly changed the output of the crack growth rate model; consequently, these factors were fixed at 0.5 throughout the modeling process.

The effective stress intensity range was unable to accurately predict the microstructurally small crack growth. It is well known that the effective stress intensity factor range can be approximately related to crack growth rate via the following relationship in Eq. (2) where an exponent of 4 has been suggested [16]. If $\Delta K_{\text{eff}}$ is set equivalent to $\Delta \sigma_{\text{eff}} \sqrt{\pi c}$, then the crack growth rate is proportional to $c^2$.

$$\frac{dc}{dN} \approx C(\Delta K_{\text{eff}})^4 \approx c^2 \quad (2)$$

Now consider the $\Delta \text{CTOD}$ versus crack growth relationship derived in Eq. (3) [16].

$$\frac{dc}{dN} \approx \Delta \text{CTOD} \approx \frac{\Delta(\Delta K^2)}{E} \approx c \quad (3)$$

For this equation, with $\Delta K$ simply equal to $\Delta \sigma \sqrt{\pi c}$, crack growth is predicted to have a linear relationship with crack length. The variables $\sigma_c$ and $E$ represent the cyclic yield strength and cyclic modulus of elasticity, respectively. The variable $\chi$ is an empirical constant. This relationship has been experimentally verified by McDowell et al. [11] to be suitable for microstructurally small crack growth predictions. Additionally, Lardner [49] found this relationship to be a suitable equation to predict crack length when considering physically small cracks. Lardner also argues that an exponent higher than 2 on $K$ is justified if high enough stresses are applied to the material. McClintock argues [50] that for crack growth rate to be linear with crack length, a single crack must solely be driven by plasticity associated with the crack tip. On the other hand, he considered an exponent greater than one on crack length to be associated with crack coalescence.

The $\Delta \text{CTOD}$ criterion was able to predict the crack growth rate from the surface replications of all tests regardless of load ratio and amount of crack coalescence. Laird [50] argues that the basis of the crack length exponent in Eq. (2) being greater than one being caused by crack coalescence is invalid because the amount of damage associated with this prediction is too high. The ability of $\Delta \text{CTOD}$ to predict crack growth has been considered to be related to grain boundary crack deflections and the roughness of the cracks themselves, where less roughness increased the crack growth rate predictions from $\Delta \text{CTOD}$ based equations [45]. However, little crack roughness was observed via replications from both $R$ values during SEM analysis. Nonetheless, the empirical fact remains that crack tip opening displacement better characterizes microstructurally small crack growth in this material.

Effective stress intensity range values were calculated from the model and are displayed in Fig. 12. While comparing the effective stress intensity factor data from Figs. 6–12, a conclusion can be made that assuming the cracks were in the microstructurally small crack growth stage was fair. If the computed $R = -1.0$ $\Delta K_{\text{eff}}$ versus crack growth rate data followed the same trend as the $R = 0.1$ data in Fig. 6, then both the computed $R = -1.0$ and $R = 0.1$ data have yet to reach the middle region of crack growth shown in Fig. 6. This provides a better foundation to show that the model was truly predicting short crack growth behavior of the material.

Fig. 13 displays the differences between the $\Delta \text{CTOD}$ based crack growth predictions and the traditional closure based crack growth predictions using the effective stress intensity factor range. As the load ratio varied from $R = 0.1$ to $R = -1.0$, the differences increased for both stress levels. While the difference for both stress levels...
was slight as the microstructurally small crack grew for $R = 0.1$ loading, the differences were larger compared for $R = -1.0$. The model output showed that crack growth rate extracted from $\Delta CTOD$ is an effective method for predicting crack growth at least in this particular orientation. More research needs to be done with regard to predicting crack growth in all orientations of this rolled plate using $\Delta CTOD$ criteria. While the model computed crack depth and crack length on the free surface, only the surface lengths were measured experimentally. If experimental data for crack depth could be made available, a more accurate calibration for the model would be possible. With regard to predicting crack growth for different orientations in the plate, the only change needed may be alterations of $G$ and compression constraint factors for different orientations.

5. Conclusions

The objective of this research was to investigate and model the microstructurally small crack growth behavior of a rolled AZ31 magnesium plate under stress controlled fatigue conditions. Crack growth rate versus stress intensity factor range data was obtained...
from CT specimens and used as input in a strip-yield model. The strip-yield model was used to compare predicted microstructurally small crack growth rates with the experimental crack growth rates of flat, reduced gage specimens. Microstructure analysis was also performed in addition to fatigue testing and modeling. Based on the experimental and computational results, the following conclusions can be made:

1. Although Repliset is able to capture cracks as small as 10–20 μm, the amount of time needed to take a sufficient amount of data for microstructurally small crack growth analysis is not very practical for the investigated rolled AZ31 magnesium.

2. Because cracks were observed to grow in a planar fashion, little to no amounts of roughness induced crack closure occurred during fatigue testing of rolled AZ31 magnesium.

3. Fatigue crack growth rate was largely independent of load ratio, R, for AK > 4.50 MPa√m when testing with CT specimens. This suggests a negligible influence from crack closure. In the threshold regime, however, crack growth rate was strongly influenced by the value of R.

4. Crack opening stresses for R = –1.0 load control tests were predicted to be nonexistent throughout the microstructurally small crack growth regime. Thus, crack closure concepts are unnecessary and crack growth can be predicted using stress intensity factor ranges alone.

5. With respect to loading taking place in the rolling direction and crack growth taking place in the transverse direction of the AZ31 magnesium rolled plate, crack tip opening displacement criterion is seemingly a more accurate way to characterize microstructurally small crack growth rates rather than the effective stress intensity factor range. Using plasticity induced crack closure concept and effective stress intensity factor range to predict microstructurally small crack growth rates provided unsatisfactory predictions.

6. The crack growth regime was found to constitute the majority of the life in almost all of the specimens used to measure microstructurally small crack growth rates.

7. In the future, a more accurate calibration can be obtained for the model if experimental crack depth data is used in conjunction with crack surface length when calibrating the model.

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